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Interface crack extension at any constant speed in orthotropic or transversely isotropic bimaterials—I. General exact solutions

L.M. Brock *

Department of Mechanical Engineering, University of Kentucky, 521 CRMS Building Lexington Campus, Lexington, KY 40506-0108, USA

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Abstract

A semi-infinite crack along the interface of two dissimilar half-spaces extends under in-plane loading. Each half-space belongs to a class of orthotropic or transversely isotropic elastic materials, the crack can extend at any constant speed, and all six possible relations between the four body wave speeds are considered. A steady dynamic situation is treated, and exact full displacement fields derived. A key step is a factorization that produces, despite anisotropy, simple solution forms and compact crack speed-dependent functions that exhibit the Rayleigh and Stoneley speeds as roots. These roots are calculated for various representative bimaterials.

Closed-form crack opening displacement gradient and interface stress fields are also derived from a general set of coupled singular integral equations. The equation eigenvalues can, depending on crack speed, be complex/imaginary conjugates, purely real, or zero. This suggests possibilities observed in other studies: oscillations and square-root singular behavior at the crack edge, non-singular behavior, singular behavior not of square-root order, and the radiation of displacement gradient discontinuities at crack speeds beyond the purely sub-sonic range.

These possibilities are explored further in terms of two important special cases in Part II of this study [Int. J. Solids Struct., 39, 1183–1198]. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The 2D study of interface cracks in perfectly bonded (welded) half-spaces of dissimilar elastic materials under in-plane loading sheds light on the failure of composites. Static analyses of stationary cracks in isotropic bimaterials (England, 1965; Erdogan, 1965; Rice and Sih, 1965) produced exact solutions by classical complex variable methods (Muskhelishvili, 1975). The stresses exhibit, as observed previously (Williams, 1959) in asymptotic results, both the standard square-root singularity (Sneddon and Lowengrub,

* Tel.: +1-859-257-2839; fax: +1-859-257-8057.
E-mail address: brock@engr.uky.edu (L.M. Brock).

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1969) and an infinite number of sign reversals in the limit as the crack edge is approached. Thus, the crack edge vicinity can undergo compression even if the applied stresses are tensile. The region for which the first sign change occurs is a small fraction (e.g. England, 1965) of the crack length. Nevertheless, models that allow crack surface sliding contact to remove the oscillatory behavior have been developed (Comninou, 1977). More recently, Ting (1990) and Ni and Nemat-Nasser (1991, 1992) examined the interface crack in equilibrium between dissimilar anisotropic materials. The latter two studies considered both the stress-free crack, and one that, after Comninou (1977), includes contact zones.

The actual crack extension process for dissimilar isotropic solids under in-plane loading has been modeled in 2D transient studies (Brock, 1976) for the case of sub-critical crack speed. Liu et al. (1995) have considered via asymptotics interface crack extension in isotropic bimaterials, with emphasis on the rigid/elastic case. The transient results are approximate, but exhibit, nevertheless, stresses with square-root singular/oscillatory behavior. The asymptotic studies demonstrate that the oscillatory behavior is maintained, but the singularity vanishes, for super-critical/sub-sonic crack extension, while the oscillations vanish and a singularity not of square-root order appears for trans-sonic crack speeds.

That trans-sonic crack speeds can occur has been demonstrated for isotropic (Freund, 1979; Broberg, 1989) and orthotropic (Broberg, 1999) solids. For isotropic bimaterials, Yu and Wang (1994) have analyzed anti-plane interface fracture for speeds that range between the constituent shear wave speeds. Indeed, the in-plane isotropic bimaterial fracture work of Liu et al. (1995) follows from other analytical and experimental efforts (Liu et al., 1993; Lambros and Rosakis, 1995), and itself has been extended (Huang et al., 1996; Huang et al., 1998; Rosakis et al., 1999; Huang and Gao, 2001).

The present article is Part I of a two-part study that complements the aforementioned work: Steady dynamic extension under in-plane loading of a semi-infinite crack along the interface of two perfectly bonded dissimilar half-spaces is considered. The half-spaces belong to a class of orthotropic or transversely isotropic materials, crack extension is at any constant speed, and all six possible relations between the four body wave speeds are considered.

Exact expressions are obtained for the displacement fields in both half-spaces, and closed-form results given for the crack-opening displacement gradients and the interface stresses. The results are examined for their consistency with the results noted above, and for the general role of anisotropy on crack extension. A key step in this regard is the factorization of certain functions of crack speed in the solution transform space that leads to cancellations of terms that induce complex branch points in the plane of the crack speed. For real crack speeds, this is of limited significance, but the cancellations also render, despite anisotropy, the solution expressions in rather simple forms.

Moreover, functions related to, but simpler in form than, the classical Rayleigh (Achenbach, 1973) and Stoneley (Cagniard, 1962) functions for isotropic elasticity arise. These allow the development of compact expressions, exact to within simple quadratures, for the Rayleigh speeds and (when it exists) the Stoneley speed. These speeds are calculated for five materials that typify the class of orthotropic or transversely isotropic materials considered. The main purpose of Part I, however, is to provide the basis for studying some important special cases of interface crack extension in Part II.

2. Basic problem

Consider two half-spaces and the Cartesian coordinates (x, y, z). The half-spaces are perfectly bonded along the plane $y = 0, x < 0$. The half-space materials are each of a class of linear homogeneous anisotropic solids whose non-trivial governing equations in plane strain in the absence of body forces have the form

$$\begin{aligned} c_{11}u_{x,xx} + c_{44}u_{x,yy} + (c_{13} + c_{44})u_{y,xy} &= \rho\ddot{u}_x \\ c_{44}u_{y,xx} + c_{33}u_{y,yy} + (c_{13} + c_{44})u_{x,xy} &= \rho\ddot{u}_y \end{aligned} \tag{1}$$

with the stress–strain equations

$$\begin{aligned}\sigma_x &= c_{11}u_{x,x} + c_{13}u_{y,y} \\ \sigma_y &= c_{13}u_{x,x} + c_{33}u_{y,y} \\ \sigma_{xy} &= c_{44}(u_{x,y} + u_{y,x})\end{aligned} \tag{2}$$

For the half-space $y > 0$, the additional subscript 1 is understood; the subscript 2 is understood for the half-space $y < 0$. These equations hold for both orthotropic and transversely isotropic materials, where the x- and y-axis are axes of material symmetry. The (u_x, u_y) are the (x, y)-components of displacement, while $(\dot{\ })$ and $(\ )_{,s}$ denote differentiation by time and a variable s, respectively. The constants $(c_{11}, c_{33}, c_{13}, c_{44})$ are a subset of the elasticities c_{ik} $(i, k = 1, 2, \ldots, 6)$ in the generalized Hooke's law (Sokolnikoff, 1983), and ρ is the mass density. Eq. (1) is a special case of a general form that involves four constants that can be linearly related to various subsets of the c_{ik} (Scott and Miklowitz, 1967). General observations on crystal structure and the c_{ik} are given by Nye (1957) and Theocaris and Sokolis (2000). In the present case, the isotropic limit for each half-space can be obtained by setting $c_{11} = c_{33} = \lambda + 2\mu$, $c_{13} = \lambda$, $c_{44} = \mu$, where the additional subscripts 1 $(y > 0)$ or 2 $(y < 0)$ are understood, and (λ, μ) are the Lamé constants.

Both half-spaces are at rest when constant shear and compressive forces (line loads in the z-direction) of magnitudes (τ, σ) are applied to opposite faces of the crack. The forces are translated toward the crack edge with constant speed v, thereby extending the crack in the positive x-direction. As depicted schematically in Fig. 1, a steady dynamic situation is achieved in which the crack edge also extends with speed v, and the translating forces remain a fixed distance L from the edge. As also indicated, it is convenient to fix the coordinates (x, y, z) to the moving crack edge, so that the interface conditions take the form

$$\sigma_{xy1} = \sigma_{xy2} = -\tau\delta(x + L), \quad \sigma_{y1} = \sigma_{y2} = -\sigma\delta(x + L) \ \ (y = 0, x < 0) \tag{3a}$$

$$u_{x1} - u_{x2} = u_{y1} - u_{y2} = \sigma_{xy1} - \sigma_{xy2} = \sigma_{y1} - \sigma_{y2} = 0 \ \ (y = 0, x > 0) \tag{3b}$$

Here δ is the Dirac function.

Because the process is one of steady plane-strain, field quantities in both half-spaces depend only on (x, y), and the operator $(\dot{\ })$ in the inertial frame can be replaced with $-v(\ )_{,x}$. For convenience, therefore, the parameters

$$\mu = c_{44}, \quad v_{\mathrm{r}} = \sqrt{\frac{c_{44}}{\rho}} \tag{4}$$

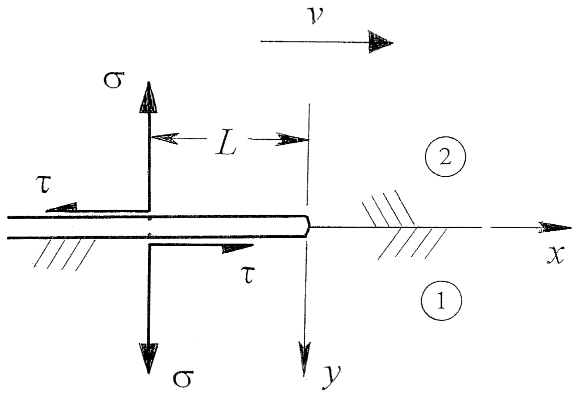

Fig. 1. Schematic of interface crack extension under in-plane loading.

are introduced, which allows the dimensionless quantities

$$\alpha = \frac{c_{33}}{\mu} \tag{5a}$$

$$\beta = \frac{c_{11}}{\mu} \tag{5b}$$

$$\gamma = 1 + \alpha\beta - m^2 \tag{5c}$$

$$m = 1 + \frac{c_{13}}{\mu} \tag{5d}$$

$$c_0 = \frac{v}{v_0} \tag{5e}$$

to be employed. The subscripts 1 or 2 are understood in Eqs. (4) and (5a)–(5d), and

$$v_0 = \min(v_{r1}, v_{r2}) \tag{6}$$

The definitions (5a)–(5d) follow from Payton (1983) and, in the isotropic limit, the v_{rk} are classical rotational wave speeds. Eqs. (1) and (2) can now be written as

$$(\beta - c^2)u_{x,xx} + u_{x,yy} + mu_{y,xy} = 0 \tag{7a}$$

$$(1 - c^2)u_{y,xx} + \alpha u_{y,yy} + mu_{x,xy} = 0 \tag{7b}$$

$$c = \frac{c_0}{n}, \quad n = \frac{v_r}{v_0} \tag{7c}$$

where the subscript 1 or 2 is understood for all quantities save (v_0, c_0, x, y), and

$$\begin{aligned} \frac{1}{\mu}\sigma_x &= \beta u_{x,x} + (m-1)u_{y,y} \\ \frac{1}{\mu}\sigma_y &= (m-1)u_{x,x} + \alpha u_{y,y} \\ \frac{1}{\mu}\sigma_{xy} &= u_{x,y} + u_{y,x} \end{aligned} \tag{8}$$

For purposes of illustration, we consider (Payton, 1983; Brock et al., 2001) the constraints

$$\begin{aligned} &2\sqrt{\alpha\beta} \leqslant \gamma \leqslant 1 + \alpha\beta \quad (1 < \beta < \alpha) \\ &\alpha + \beta \leqslant \gamma \leqslant 1 + \alpha\beta \quad (1 < \alpha < \beta) \\ &2\alpha \leqslant \gamma \leqslant 1 + \alpha^2 \quad (1 < \beta = \alpha) \end{aligned} \tag{9}$$

The class of materials governed by Eq. (9) includes beryl, cobalt, ice, magnesium and titanium, as well as the isotropic limit. In addition, the displacements and their gradients are expected to vanish when $(x^2 + y^2)^{1/2} \to \infty$, and be non-singular almost everywhere, except at sites such as the crack edge $(x, y) = 0$ and load positions $x = -L$, $y = 0\pm$.

3. Relative displacement solution

It is convenient to first consider a problem governed by Eqs. (6)–(9) and the boundedness/continuity requirements, but with Eqs. (3a) and (3b) replaced by the unmixed interface conditions

$$\sigma_{xy1} - \sigma_{xy2} = \sigma_{y1} - \sigma_{y2} = 0, \quad u_{x1} - u_{x2} = U(x), \quad u_{y1} - u_{y2} = V(x) \ \ (y = 0) \tag{10}$$

Here (U, V) are the relative tangential and normal crack face displacements, i.e. the crack-opening displacements, and so must vanish identically for $x > 0$, and be continuous at $x = 0$. To solve this related problem, the bilateral Laplace transform (van der Pol and Bremmer, 1950) and its inverse operator are introduced:

$$\hat{F} = \int_{-\infty}^{\infty} F(x)\mathrm{e}^{-px}\,\mathrm{d}x \tag{11a}$$

$$F(x) = \frac{1}{2\pi i}\int \hat{F}\mathrm{e}^{px}\,\mathrm{d}p \tag{11b}$$

In Eq. (11a) p is imaginary, and integration in Eq. (11b) is along a Bromwich contour in the p-plane. Application of Eq. (11a) to Eqs. (7a)–(7c), (8) and (10) gives a coupled set of second-order differential equations in y with boundary conditions. This can be solved to give

$$\frac{1}{\mu_l}\hat{u}_x = \frac{mB\mathrm{e}^{-a|y|\sqrt{p}\sqrt{-p}}}{\psi(b-a)S}\left(A_U\hat{U} + \frac{\sqrt{-p}}{\sqrt{p}}A_V\hat{V}\right) + \frac{\mathrm{e}^{-b|y|\sqrt{p}\sqrt{-p}}}{B(b-a)S}\left(-B_U\hat{U} + \frac{\sqrt{-p}}{\sqrt{p}}B_V\hat{V}\right) \tag{12a}$$

$$\frac{1}{\mu_l}\hat{u}_y = \frac{a\mathrm{e}^{-a|y|\sqrt{p}\sqrt{-p}}}{B(b-a)S}\left(-\frac{\sqrt{-p}}{\sqrt{p}}A_U + A_V\hat{V}\right) + \frac{mb\mathrm{e}^{-b|y|\sqrt{p}\sqrt{-p}}}{\psi B(b-a)S}\left(\frac{\sqrt{-p}}{\sqrt{p}}B_U\hat{U} + B_V\hat{V}\right) \tag{12b}$$

$$(\hat{U}, \hat{V}) = \int_C (U, V)\mathrm{e}^{-pt}\,\mathrm{d}t \tag{12c}$$

Here C attached to an integral signifies integration over the real interval $(-\infty, 0)$, and the subscript $k = (1, 2)$ is understood on all terms save $(\mu_l, p, \hat{U}, \hat{V}, S)$, while the subscript $l = (2, 1)$ when $k = (1, 2)$. For half-space 1 $(y > 0)$,

$$\begin{bmatrix} A_U \\ A_V \end{bmatrix} = \begin{bmatrix} -b_1M_1Q_1 - P_1A_2(a+b)_2 & m_1b_1R_2 \\ b_1Q_1\alpha_2B_2(a+b)_2 + P_1M_2 & -\psi_1R_2 \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \tag{13a}$$

$$\begin{bmatrix} B_U \\ B_V \end{bmatrix} = \begin{bmatrix} -a_1P_1M_2 - B_1^2Q_1A_2(a+b)_2 & a_1\psi_1R_2 \\ -B_1^2Q_1M_2 - a_1P_1\alpha_2B_2(a+b)_2 & m_1B_1^2R_2 \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \tag{13b}$$

while for half-space 2 $(y < 0)$,

$$\begin{bmatrix} -A_U \\ A_V \end{bmatrix} = \begin{bmatrix} -b_2Q_2M_1 - P_2A_1(a+b)_1 & m_2b_2R_1 \\ b_2Q_2\alpha_1B_1(a+b)_1 + P_2M_1 & -\psi_2R_1 \end{bmatrix}\begin{bmatrix} \mu_2 \\ \mu_1 \end{bmatrix}$$
$$\begin{bmatrix} B_U \\ -B_V \end{bmatrix} = \begin{bmatrix} -a_2P_2M_1 - B_2^2Q_2A_1(a+b)_1 & a_2\psi_2R_1 \\ -B_2^2Q_2M_1 - a_2P_2\alpha_1B_1(a+b)_1 & m_2B_2^2R_1 \end{bmatrix}\begin{bmatrix} \mu_2 \\ \mu_1 \end{bmatrix} \tag{14}$$

In Eqs. (12a)–(12c) and (14) the definitions

$$P = \psi + mB^2, \quad Q = m + \psi \tag{15a}$$

$$\phi = \alpha a^2 - A^2, \quad \psi = \alpha b^2 - B^2, \quad \Omega = \alpha a^2 - B^2 \tag{15b}$$

$$M = A + (1-m)B, \quad R = c^2 A + BC, \quad C = (m-1)^2 - A^2 \tag{15c}$$

hold, where the subscript $k = (1, 2)$ and

$$\sqrt{\alpha}(a, b) = \frac{1}{\sqrt{2}}\sqrt{T \pm \sqrt{T^2 - 4A^2B^2}} = \frac{1}{2}\left(\sqrt{T + 2AB} \pm \sqrt{T - 2AB}\right) \tag{16a}$$

$$\sqrt{\alpha}(a \pm b) = \sqrt{A \pm B + m}\sqrt{A \pm B - m} \tag{16b}$$

$$T = A^2 + B^2 - m^2, \quad A = \sqrt{\alpha}\sqrt{\beta - c^2}, \quad B = \sqrt{1 - c^2} \tag{16c}$$

The non-subscripted denominator term S in Eqs. (12a) and (12b) is given by

$$S = \mu_1\mu_2[2M_1M_2 + (a+b)_1(a+b)_2(\alpha_1B_1A_2 + \alpha_2B_2A_1)] - \mu_1^2(A+B)_2R_1 - \mu_2^2(A+B)_1R_2 \tag{17}$$

The transform solutions to the related problem are seen to be functions of $\sqrt{\pm p}$ and coefficients that are combinations of quantities that are themselves functions of the dimensionless crack speed c_0, elasticities μ_k and dimensionless material constants $(\alpha_k, \beta_k, \gamma_k, m_k)$. Introduction of the branch cuts $\mathrm{Im}(p) = 0$, $\mathrm{Re}(p) < 0$ and $\mathrm{Im}(p) = 0$, $\mathrm{Re}(p) > 0$ for $\sqrt{\pm p}$, respectively, guarantees that $\mathrm{Re}(\sqrt{p}\sqrt{-p}) \geqslant 0$ in the cut p-plane. Therefore, for c_0 such that (a_k, b_k) are real and positive, boundedness of Eqs. (12a) and (12b) as $|y| \to \infty$ is assured.

4. Behavior of crack speed-dependent functions

In this light, introduction of the branch cuts $\mathrm{Im}(c_0) = 0$, $|\mathrm{Re}(c_0)| > n_k\sqrt{\beta_k}$ and $\mathrm{Im}(c_0) = 0$, $|\mathrm{Re}(c_0)| > n_k$ for (A_k, B_k), respectively, guarantees that $\mathrm{Re}(A_k, B_k) \geqslant 0$ in a cut c_0-plane. The branch points $(n_k, n_k\sqrt{\beta_k})$ correspond to, respectively, the non-dimensionalized rotational and dilatational wave speeds in the two half-spaces associated with the x-axis of material symmetry. In light of Eq. (9), the six general relations

$$n_1 < n_1\sqrt{\beta_1} < n_2 < n_2\sqrt{\beta_2}, \quad n_1 < n_2 < n_1\sqrt{\beta_1} < n_2\sqrt{\beta_2}, \quad n_1 < n_2 < n_2\sqrt{\beta_2} < n_1\sqrt{\beta_1} \tag{18a–c}$$

$$n_2 < n_2\sqrt{\beta_2} < n_1 < n_1\sqrt{\beta_1}, \quad n_2 < n_1 < n_2\sqrt{\beta_2} < n_1\sqrt{\beta_1}, \quad n_2 < n_1 < n_1\sqrt{\beta_1} < n_2\sqrt{\beta_2} \tag{18d–f}$$

are possible, with equalities following as special cases. These relations correspond to those for isotropic materials (Cagniard, 1962).

The quantities (a_k, b_k) share the branch cuts of (A_k, B_k), respectively, but Eq. (9) allows the additional complex branch points defined by Eq. (7c) and

$$(\alpha_k - 1)^2c_k^2 = \gamma_k(1 + \alpha_k) - 2\alpha_k(1 + \beta_k) \pm i2m_k\sqrt{\alpha_k}\sqrt{\gamma_k - \alpha_k - \beta_k} \;\; (k = 1, 2) \tag{19}$$

in the c_0-plane. These are also roots of $b_k^2 - a_k^2$ and, indeed, the forms of Eqs. (12a)–(12c) resulted by extracting the product $(b_1 - a_1)(b_2 - a_2)$ and the factor $b_l - a_l$ from, respectively, the denominator and numerators of the expressions for $(\hat{u}_x, \hat{u}_y)$ that were originally derived. Cancellation, therefore, produced the denominator factor $b_k - a_k$ indicated in Eqs. (12a) and (12b). However, the exponential terms in Eqs. (12a)–(12c) become the same when $b_k = a_k$, and use of Eqs. (15a)–(15c) and (16a)–(16c) and the related formulas

$$\alpha a^2\psi + B^2\phi = 0, \quad \alpha ab = AB, \quad \phi + \psi + m^2 = 0 \tag{20}$$

with subscripts $k = (1, 2)$ understood, shows that the resulting coefficients of the terms $(\hat{U}, \hat{V})/(b-a)_k$ also behave as $b_k - a_k \to 0$. That is, the terms $(b_1 - a_1, b_2 - a_2)$ effectively cancel from the solution transforms.

By then allowing $a_k + b_k$, given by Eq. (16b), to be continuous across cuts associated with Eq. (19), even though $(a_k, b_k, b_k - a_k)$ remain multi-valued there, the transforms Eqs. (12a)–(12c) exhibit only the branch cuts of (A_1, B_1, A_2, B_2). This feature becomes useful when crack speeds are such that $v > v_0$.

In any case, the factorization simplifies the solution transforms, and produces the functions (R_k, S) defined in Eqs. (15c) and (17), respectively. The R_k are analytic in the c_0-plane cut along $\mathrm{Im}(c_0) = 0$, $|\mathrm{Re}(c_0)| > n_k$ and exhibit real roots that, in view of Eq. (16b), give the values $c_k = c_{\mathrm{R}k}(0 < c_{\mathrm{R}k} < 1)$. Indeed, rationalization of the formula $R_k = 0$ gives a cubic equation in c_0^2 that corresponds exactly to one obtained by Payton (1983) for the roots of a transversely isotropic Rayleigh function. That is, $v_{\mathrm{R}k} = (c_\mathrm{R} v_\mathrm{r})_k$ is the Rayleigh wave speed parallel to the x-axis of material symmetry for the material class considered here. The term R_k is, despite being simpler in form than its isotropic counterpart, e.g. Achenbach (1973), the effective Rayleigh function. As an alternative to the cubic equation solution, $c_{\mathrm{R}k}$ can be, by following a general approach (Brock, 1998), obtained exactly to within a simple quadrature as

$$c_\mathrm{R} = \sqrt{\frac{\alpha\beta - (m-1)^2}{\sqrt{\alpha\beta}(1+\sqrt{\alpha})}\frac{1}{G_\mathrm{R}}}, \quad \ln G_\mathrm{R} = -\frac{1}{\pi}\int_1^{\sqrt{\beta}} \tan^{-1}\frac{C\sqrt{t^2-1}}{t^2\sqrt{\alpha}\sqrt{\beta - t^2}}\frac{\mathrm{d}t}{t} \tag{21}$$

Here the subscript 1 or 2 is understood, and Eq. (9) guarantees that the coefficient of G_R is real and positive.

The function S has the branch cuts $\mathrm{Im}(c_0) = 0, n_0 < |\mathrm{Re}(c_0)| < n^*\sqrt{\beta^*}$, where

$$n_0 = \min(n_k), \quad n^*\sqrt{\beta^*} = \max(n_k\sqrt{\beta_k}) \tag{22}$$

and $k = (1, 2)$. It can be shown that when

$$2M_l + \alpha_l B_l (a+b)_l \sqrt{\beta_k - 1 - \frac{m_k^2}{\alpha_k}} < \frac{\mu_k}{\mu_l}(A+B)_l + \frac{\mu_l}{\mu_k}R_l \quad (c_0 = 1, v_0 = v_{\mathrm{r}k}) \tag{23}$$

where $k = (1, 2)$ and $l = (2, 1)$, S exhibits the isolated real roots $c_0 = \pm c_\mathrm{S}$ $(0 < c_\mathrm{S} < n_0)$.

That is, $v_\mathrm{S} = c_\mathrm{S} v_0$ is the Stoneley wave speed (Cagniard, 1962) associated with the x-axis of material symmetry. By a procedure analogous to that used for Eq. (21), a formula

$$c_\mathrm{S} = \frac{\sqrt{S_0}}{\sqrt{\sqrt{\beta_1\beta_2}(1+\sqrt{\alpha_1})(1+\sqrt{\alpha_2})\sqrt{(\mu_1 n_1 + \mu_2 n_2)(\mu_1 n_1\sqrt{\alpha_1} + \mu_2 n_2\sqrt{\alpha_2})}}}\frac{1}{G_\mathrm{S}} \tag{24}$$

emerges that is exact to within simple quadrature. In Eq. (24)

$$\begin{aligned} S_0 = {} & \mu_1\mu_2(\sqrt{\alpha_1\beta_2} + \sqrt{\alpha_2\beta_1})\sqrt{(\sqrt{\alpha\beta}+1)_1^2 - m_1^2}\sqrt{(\sqrt{\alpha\beta}+1)_2^2 - m_2^2} + \mu_1(\sqrt{\alpha\beta}+1-m)_1 \\ & \times [\mu_2(\sqrt{\alpha\beta}+1-m)_2 + \mu_1(\sqrt{\alpha\beta}+1)_1(\sqrt{\alpha\beta}+m-1)_1] + \mu_2(\sqrt{\alpha\beta}+1-m)_2[\mu_1(\sqrt{\alpha\beta}+1-m)_1 \\ & + \mu_2(\sqrt{\alpha\beta}+1)_2(\sqrt{\alpha\beta}+m-1)_2] \end{aligned} \tag{25}$$

where Eq. (9) guarantees that the right-hand side is real and positive. Expressions for G_S for all the cases Eqs. (18a–c) and (18d–f) when Eq. (23) is satisfied are given in Appendix A. Those results involve Eqs. (15a)–(15c) and (16a)–(16c) and the additional definitions

$$\begin{aligned} & M' = A' + (1-m)B', \quad R' = c^2A' + B'C \\ & A' = \sqrt{\alpha}\sqrt{c^2 - \beta}, \quad B' = \sqrt{c^2-1} \\ & \sqrt{2\alpha}(a', b') = \sqrt{-T \mp \sqrt{T^2 - 4A^2B^2}} \end{aligned} \tag{26}$$

In Eq. (26) the subscript 1 or 2 is understood on all quantities. The appearance of the Rayleigh function in Eq. (23) indicates that, should the Stoneley speed exist, it at least exceeds the Rayleigh speed corresponding to the half-space with the lower rotational speed, i.e.

$$v_{r1} < v_{r2} : c_S v_0 > (c_R v_r)_1 \quad v_{r2} < v_{r1} : c_S v_0 > (c_R v_r)_2 \tag{27}$$

5. Transform inversion

For crack extension that is sub-sonic with respect to both half-spaces $(0 < v < v_0)$ the quantities (A_k, a_k, B_k, b_k) are real and positive and, as noted, boundedness of Eqs. (12a)–(12c) as $|y| \to \infty$ is assured. Therefore, inversion of Eqs. (12a)–(12c) reduces to, in light of Eq. (11b), the two integrals

$$\frac{1}{2\pi i} \int \left(1, \frac{\sqrt{-p}}{\sqrt{p}}\right) e^{p(x-t) - q\sqrt{p}\sqrt{-p}} \, dp \quad (q = a_k|y|, b_k|y|) \tag{28}$$

The analyticity of the right-hand sides of Eqs. (12a) and (12b) implies that the entire Im(p)-axis in Eq. (28) can serve as the Bromwich contour. Standard tables (Peierce and Foster, 1956) can be used to show that the integrals give

$$\frac{1}{\pi} \frac{(q, x-t)}{(t-x)^2 + q^2} \tag{29}$$

Eqs. (12a) and (12b) then yield for $0 < v < v_0$

$$\begin{aligned} u_x &= \frac{\mu_l m B}{\psi(b-a) S\pi} \int_C \frac{dt}{(t-x)^2 + a^2 y^2} [A_U U(t) a|y| - A_V V(t)(t-x)] \\ &\quad - \frac{\mu_l}{B(b-a) S\pi} \int_C \frac{dt}{(t-x)^2 + b^2 y^2} [B_U U(t) b|y| + B_V V(t)(t-x)] \\ u_y &= \frac{\mu_l a}{B(b-a) S\pi} \int_C \frac{dt}{(t-x)^2 + a^2 y^2} [A_U U(t)(t-x) + A_V V(t) a|y|] \\ &\quad + \frac{\mu_l m b}{\psi B(b-a) S\pi} \int_C \frac{dt}{(t-x)^2 + b^2 y^2} [-B_U U(t)(t-x) + B_V V(t) b|y|] \end{aligned} \tag{30}$$

The subscript $k = (1, 2)$ is understood on all terms save $(\mu_l, S, x, y, t, U, V)$, and the subscript $l = (2, 1)$.

For crack speeds $v > v_0$, however, various elements of (A_k, a_k, B_k, b_k) are imaginary, i.e. Eq. (26) comes into play, and the exponential coefficients in Eqs. (12a)–(12c) may become complex. It is therefore convenient to define for $c_0 \pm i0$ the real and imaginary parts of the Stoneley term S as

$$S = R_S \mp i I_S, \quad \omega = \frac{I_S}{R_S} \tag{31}$$

and the real and imaginary parts of the other factors in Eqs. (12a)–(12c) as

$$\begin{aligned} &\frac{mB}{\psi(b-a)} \begin{bmatrix} A_U \\ A_V \end{bmatrix} = \begin{bmatrix} R_A^U \mp i I_A^U \\ R_A^V \mp i I_A^V \end{bmatrix}, \quad \frac{a}{B(b-a)} \begin{bmatrix} -A_U \\ A_V \end{bmatrix} = \begin{bmatrix} R_C^U \mp i I_C^U \\ R_C^V \mp i I_C^V \end{bmatrix} \\ &\frac{1}{B(b-a)} \begin{bmatrix} -B_U \\ B_V \end{bmatrix} = \begin{bmatrix} R_B^U \mp i I_B^U \\ R_B^V \mp i I_B^V \end{bmatrix}, \quad \frac{mb}{\psi B(b-a)} \begin{bmatrix} B_U \\ B_V \end{bmatrix} = \begin{bmatrix} R_D^U \mp i I_D^U \\ R_D^V \mp i I_D^V \end{bmatrix} \end{aligned} \tag{32}$$

In Eq. (32) the subscript $k = (1,2)$ is understood for all terms. General results needed to identify these real and imaginary parts when $v > v_0$ for all the cases Eqs. (18a–c) and (18d–f) for either half-space 1 or half-space 2 are given in Appendix B.

In light of these results, it can be shown that Eqs. (12a)–(12c) again gives transforms that are bounded for $|y| \to \infty$, and that when crack speeds are sub-sonic with respect to the half-space k $(0 < v < v_{rk})$, application of Eq. (11b) gives for that half-space

$$\begin{aligned}
u_x &= \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + a^2y^2}\{[(R_A^U + \omega I_A^U)a|y| - (I_A^U - \omega R_A^U)(t-x)]U(t) - [(I_A^V - \omega R_A^V)a|y| \\
&\quad + (R_A^V + \omega I_A^V)(t-x)]V(t)\} + \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + b^2y^2}\{[(R_B^U + \omega I_B^U)b|y| - (I_B^U - \omega R_B^U)(t-x)] \\
&\quad \times U(t) - [(I_B^V - \omega R_B^V)b|y| + (R_B^V + \omega I_B^V)(t-x)]V(t)\} \\
u_y &= \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + a^2y^2}\{-[(I_C^U - \omega R_C^U)a|y| + (R_C^U + \omega I_C^U)(t-x)]U(t) + [(R_C^V + \omega I_C^V)a|y| \\
&\quad - (I_C^V - \omega R_C^V)(t-x)]V(t)\} + \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + b^2y^2}\{-[(I_D^U - \omega R_D^U)b|y| + (R_D^U + \omega I_D^U)(t-x)] \\
&\quad \times U(t) + [(R_D^V + \omega I_D^V)b|y| - (I_D^V - \omega R_D^V)(t-x)]V(t)\} \\
F_l &= \frac{\mu_l}{R_S(1+\omega^2)}
\end{aligned} \tag{33}$$

Here the subscript $k = (1,2)$ is understood and $l = (2,1)$, and the result (30) falls out as the special case $v_0 = v_{rk}$. For crack speeds that lie in the trans-sonic range for the half-space k $(v_{rk} < v < \sqrt{\beta_k}v_{rk})$, the displacements for that half-space are

$$\begin{aligned}
u_x &= \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + a^2y^2}\{[(R_A^U + \omega I_A^U)a|y| - (I_A^U - \omega R_A^U)(t-x)]U(t) - [(I_A^V - \omega R_A^V)a|y| \\
&\quad + (R_A^V + \omega I_A^V)(t-x)]V(t)\} - \frac{F_l}{\pi}\int_C \frac{dt}{t-x-b'|y|}[(I_B^U - \omega R_B^U)U(t) + (R_A^V + \omega I_A^V)V(t)] \\
&\quad + F_l[(R_B^U + \omega I_B^U)U(x+b'|y|) - (I_B^V - \omega R_B^V)V(x+b'|y|)] \\
u_y &= \frac{F_l}{\pi}\int_C \frac{dt}{(t-x)^2 + a^2y^2}\{-[(I_C^U - \omega R_C^U)a|y| + (R_C^U + \omega I_C^U)(t-x)]U(t) + [(R_C^V + \omega I_C^V)a|y| \\
&\quad - (I_C^V - \omega R_C^V)(t-x)]V(t)\} - \frac{F_l}{\pi}\int_C \frac{dt}{t-x-b'|y|}[(R_D^U + \omega I_D^U)U(t) + (I_D^V - \omega R_D^V)V(t)] \\
&\quad + F_l[-(I_D^U - \omega R_D^U)U(x+b'|y|) + (R_D^V + \omega I_D^V)V(x+b'|y|)]
\end{aligned} \tag{34}$$

Again, the subscript $k = (1,2)$ is understood, and $l = (2,1)$. It is also understood that the non-integral terms do not appear unless $x + b'|y| < 0$, and that the integrals involving b' must then be interpreted in the Cauchy principal value sense. When crack speeds are super-sonic with respect to a half-space k $(v > \sqrt{\beta_k}v_{rk})$, the displacements for that half-space are

$$\begin{aligned} u_x = &-\frac{F_l}{\pi}\int_C \frac{dt}{t-x-a'|y|}[(I_A^U-\omega R_A^U)U(t)+(R_A^V+\omega I_A^V)V(t)]+F_l[(R_A^U+\omega I_A^U)U(x+a'|y|) \\ &-(I_A^V-\omega R_A^V)V(x+a'|y|)]-\frac{F_l}{\pi}\int_C \frac{dt}{t-x-b'|y|}[(I_B^U-\omega R_B^U)U(t)+(R_B^V+\omega I_B^V)V(t)] \\ &+F_l[(R_B^U+\omega I_B^U)U(x+b'|y|)-(I_B^V-\omega R_B^V)V(x+b'|y|)] \\ u_y = &-\frac{F_l}{\pi}\int_C \frac{dt}{t-x-a'|y|}[(R_C^U+\omega I_C^U)U(t)+(I_C^V-\omega R_C^V)V(t)]+F_l[-(I_C^U-\omega R_C^U)U(x+a'|y|) \\ &+(R_C^V+\omega I_C^V)V(x+a'|y|)]-\frac{F_l}{\pi}\int_C \frac{dt}{t-x-b'|y|}[(R_D^U+\omega I_D^U)U(t)+(I_D^V-\omega R_D^V)V(t)] \\ &+F_l[-(I_D^U-\omega R_D^U)U(x+b'|y|)+(R_D^V+\omega I_D^V)V(x+b'|y|)] \end{aligned} \tag{35}$$

In this instance, the first and second set of non-integral terms in each displacement do not appear unless, respectively, $x+a'|y|<0$ and $x+b'|y|<0$. Then, the corresponding a'—and b'—integrals must be taken in the Cauchy principal value sense. For the case where crack speeds are super-sonic with respect to both half-spaces, Eq. (35) reduce to

$$\begin{aligned} u_x &= \frac{\mu_l}{S'}[R_A^U U(x+a'|y|)-I_A^V V(x+a'|y|)]-\frac{\mu_l}{S'\pi}\int_C \frac{dt}{t-x-b'|y|}[I_B^U U(t)+R_B^V V(t)] \\ u_y &= -\frac{\mu_l}{S'\pi}\int_C \frac{dt}{t-x-a'|y|}[R_C^U U(t)+I_C^V V(t)]+\int_C \frac{dt}{t-x-b'|y|}[R_D^U U(t)+I_D^V V(t)] \\ S' &= \mu_1\mu_2[(a'+b')_1(a'+b')_2(\alpha_1 B_1' A_2'+\alpha_2 B_2' A_1')-2M_1'M_2']+\mu_1^2(A'+B')_2R_1'+\mu_2^2(A'+B')_1R_2' \end{aligned} \tag{36}$$

In both Eqs. (35) and (36), the subscript $k=(1,2)$ is understood, and $l=(2,1)$.

It can be shown that Eqs. (33) and (34) are continuous when $v=v_{rk}$, and Eqs. (34) and (35) are continuous when $v=\sqrt{\beta_k}v_{rk}$. A key tool in the demonstration is the standard (Carrier and Pearson, 1988) relation

$$\frac{q}{(t-x)^2+q^2}\to\pi\delta(t-x)\quad(q\to 0+) \tag{37}$$

Eqs. (31) and (32) and the results of Appendix B show that the real and imaginary parts that appear in Eqs. (33)–(36) are also continuous for all positive values of v. Therefore, discontinuities in displacements that occur as the crack speed v traverses the ranges indicated in Eqs. (18a–c) and (18d–f) will arise from the behavior of the crack-opening displacement functions (U,V).

In this regard, it is seen that the non-integral terms in the regions $(x+a'|y|<0, x+b'|y|<0)$ seen in Eqs. (34)–(36) admit the possibility of lines of discontinuity $(x+a'|y|=0, x+b'|y|=0)$ in displacement gradients that radiate from the moving crack edge in a half-space k for crack speeds that exceed the sub-sonic range for that half-space. This confirms behavior noted via asymptotics for trans-sonic interface crack extension in an isotropic bimaterial (Liu et al., 1995).

6. General solution

The results (30) and (33)–(36) represent solutions to the unmixed boundary value problem characterized by Eq. (10). These will also be the interface crack problem solutions if functions (U,V) can be found such that the crack surface load conditions (3a) can be satisfied. Combining Eqs. (30) and (33)–(36) with Eq. (8), setting $y=0$ in view of Eq. (37) and then substituting into Eq. (3a) gives the coupled equations

$$(R_N + \omega I_N)\frac{\mathrm{d}U}{\mathrm{d}x} - (I_N - \omega R_N)\frac{1}{\pi}⨍_C \frac{\mathrm{d}U}{\mathrm{d}t}\frac{\mathrm{d}t}{t-x} - (I_B - \omega R_B)\frac{\mathrm{d}V}{\mathrm{d}x} - (R_B + \omega I_B)\frac{1}{\pi}⨍_C \frac{\mathrm{d}V}{\mathrm{d}t}\frac{\mathrm{d}t}{t-x}$$
$$= -\frac{R_S(1+\omega^2)}{\mu_1\mu_2}\sigma\delta(x+L) \tag{38a}$$

$$-(I_A - \omega R_A)\frac{\mathrm{d}U}{\mathrm{d}x} - (R_A + \omega I_A)\frac{1}{\pi}⨍_C \frac{\mathrm{d}U}{\mathrm{d}t}\frac{\mathrm{d}t}{t-x} - (R_N + \omega I_N)\frac{\mathrm{d}V}{\mathrm{d}x} + (I_N - \omega R_N)\frac{1}{\pi}⨍_C \frac{\mathrm{d}V}{\mathrm{d}t}\frac{dt}{t-x}$$
$$= -\frac{R_S(1+\omega^2)}{\mu_1\mu_2}\tau\delta(x+L) \tag{38b}$$

for the crack-opening displacement gradients $(\mathrm{d}U/\mathrm{d}x, \mathrm{d}V/\mathrm{d}x)$ when $y = 0$, $x < 0$, where no additional subscripting is implied. Because a steady dynamic solution can be determined only to within an arbitrary rigid-body motion, obtaining the gradients is sufficient. In Eq. (38), $⨍$ denotes Cauchy principal value integration, and (R_S, ω) are defined by Eq. (31). The other integral coefficient terms are defined for $c_0 \pm \mathrm{i}0$ as real and imaginary parts, i.e.

$$\begin{aligned} R_N \mp \mathrm{i}I_N &= N = \mu_2 R_2 M_1 - \mu_1 R_1 M_2 \\ R_A \mp \mathrm{i}I_A &= N_A = \mu_2 R_2 A_1 (a+b)_1 + \mu_1 R_1 A_2 (a+b)_2 \\ R_B \mp \mathrm{i}I_B &= N_B = \mu_2 R_2 \alpha_1 B_1 (a+b)_1 + \mu_1 R_1 \alpha_2 B_2 (a+b)_2 \end{aligned} \tag{39}$$

where the definitions (15a)–(15c), (16a)–(16c) and (26) govern. The relative simplicity of Eq. (38) when compared with Eqs. (30) and (33)–(36) follows from the cancellation of terms proportional to $(b_1 - a_1, b_2 - a_2)$ that occurs explicitly when $y = 0$. For crack speeds that are sub-sonic with respect to both half-spaces $(0 < v < v_0)$, the quantities (N, N_A, N_B) are purely real. For $v > v_0$, the formulas necessary to determine the real and imaginary parts indicated in Eq. (39) are given in Appendix C.

Eqs. (38a,b) are a degenerate case of standard (Erdogan, 1976) coupled Cauchy singular integral equations. By following a version (Brock, 1999) of typical procedures for such equations, they yield

$$\begin{aligned} \frac{\mathrm{d}U}{\mathrm{d}x} &= \frac{\tau R_S}{\mu_1\mu_2}\sum_{\pm}(I_B\xi - R_B\eta)\left(\frac{\pm 1}{\chi}\right)\left[\delta(x+L)\cos\pi\upsilon - \left(\frac{-x}{L}\right)^{\upsilon}\frac{\sin\pi\upsilon}{\pi(x+L)}\right] \\ &+ \frac{\sigma R_S}{\mu_1\mu_2}\sum_{\pm}(R_N\xi + I_N\eta)\left(\frac{\mp 1}{\chi}\right)\left[\delta(x+L)\cos\pi\upsilon - \left(\frac{-x}{L}\right)^{\upsilon}\frac{\sin\pi\upsilon}{\pi(x+L)}\right] \\ \frac{\mathrm{d}V}{\mathrm{d}x} &= \frac{\sigma R_S}{\mu_1\mu_2}\sum_{\pm}(I_A\xi - R_A\eta)\left(\frac{\pm 1}{\chi}\right)\left[\delta(x+L)\cos\pi\upsilon - \left(\frac{-x}{L}\right)^{\upsilon}\frac{\sin\pi\upsilon}{\pi(x+L)}\right] \\ &+ \frac{\tau R_S}{\mu_1\mu_2}\sum_{\pm}(R_N\xi + I_N\eta)\left(\frac{\pm 1}{\chi}\right)\left[\delta(x+L)\cos\pi\upsilon - \left(\frac{-x}{L}\right)^{\upsilon}\frac{\sin\pi\upsilon}{\pi(x+L)}\right] \end{aligned} \tag{40}$$

for $x < 0$. Here summation is understood to be on the parameters $(\xi_\pm, \eta_\pm, \upsilon_\pm)$, where

$$\xi_\pm = \omega\cos\pi\upsilon_\pm + \sin\pi\upsilon_\pm, \quad \eta_\pm = \omega\sin\pi\upsilon_\pm - \cos\pi\upsilon_\pm \tag{41a}$$

$$\upsilon_\pm = \frac{1}{\pi}\tan^{-1}\frac{2A + \omega(B \pm \chi)}{2\omega A - B \mp \chi} - \frac{1}{2} \tag{41b}$$

$$\cos\pi\upsilon_\pm = \frac{2A + \omega(B \pm \chi)}{\sqrt{1+\omega^2}\sqrt{4A^2 + (B \pm \chi)^2}}, \quad \sin\pi\upsilon_\pm = -\frac{2\omega A - B \mp \chi}{\sqrt{1+\omega^2}\sqrt{4A^2 + (B \pm \chi)^2}} \tag{41c}$$

In Eq. (40) and Eqs. (41a)–(41c) the definitions

$$A = R_N^2 + I_A I_B, \quad B = 2R_N I_N - R_A I_B - R_B I_A$$
$$\chi = \sqrt{C_0^2 - 4A_0^2 B_0^2} \tag{42}$$
$$A_0 = R_A R_N + I_A I_N, \quad B_0 = R_B R_N + I_B I_N, \quad C_0 = R_A I_B - R_B I_A$$

hold, with no additional subscripting implied. Eq. (41b) give the eigenvalues of Eq. (38), and examination of Eqs. (41a)–(41c) and (42) shows that, while $(A, B, \omega, A_0, B_0, C_0)$ are real, the quantity χ could be imaginary. In that instance $(\xi_\pm, \eta_\pm, \upsilon_\pm)$ are complex conjugate pairs, so that Eq. (40) is still real-valued.

For $y = 0$, $x > 0$ the formulas

$$\sigma_{yi} = \frac{-\mu_1\mu_2}{R_S(1+\omega^2)\pi}\left[(I_N - \omega R_N)\int_C \frac{dU}{dt}\frac{dt}{t-x} + (R_B + \omega I_B)\int_C \frac{dV}{dt}\frac{dt}{t-x}\right]$$
$$\sigma_{xyi} = \frac{\mu_1\mu_2}{R_S(1+\omega^2)\pi}\left[-(R_A + \omega I_A)\int_C \frac{dU}{dt}\frac{dt}{t-x} + (I_N - \omega R_N)\int_C \frac{dV}{dt}\frac{dt}{t-x}\right] \tag{43}$$

for the interface stresses can be obtained from Eqs. (18a–c), (18d–f), (30), (33)–(36) and (40). Substitution of Eq. (43) and use of Cauchy integral theory after Brock (1999) produces

$$\sigma_{yi} = \frac{\sigma}{2\pi(x+L)}\sum_\pm\left(1 \mp \frac{C_0}{\chi}\right)\left(\frac{x}{L}\right)^\upsilon \sin\pi\upsilon + \frac{\tau B_0}{2\pi(x+L)}\sum_\pm\left(\frac{x}{L}\right)^\upsilon\left(\mp\frac{\sin\pi\upsilon}{\chi}\right)$$
$$\sigma_{xyi} = \frac{\tau}{2\pi(x+L)}\sum_\pm\left(1 \pm \frac{C_0}{\chi}\right)\left(\frac{x}{L}\right)^\upsilon \sin\pi\upsilon + \frac{\sigma A_0}{2\pi(x+L)}\sum_\pm\left(\frac{x}{L}\right)^\upsilon\left(\mp\frac{\sin\pi\upsilon}{\chi}\right) \tag{44}$$

Here summation is implied on the eigenvalues $\upsilon_\pm$, and again, the results are purely real even when χ takes on imaginary values.

Complex eigenvalues are found in static analyses of interface cracks in isotropic (England, 1965; Erdogan, 1965; Rice and Sih, 1965) and anisotropic (Ting, 1990; Ni and Nemat-Nasser, 1991, 1992) bimaterials, and in transient (Brock, 1976) and asymptotic dynamic (Liu et al., 1995; Huang et al., 1996) studies of isotropic bimaterials. They give rise to oscillations in field quantities near the crack edge. In the dynamic results the eigenvalues were dependent on crack speed: Complex eigenvalues occurred in both studies for sub-critical crack speeds, i.e. below the values of any body wave, Rayleigh or Stoneley speeds. Liu et al. (1995) found that the eigenvalues may be purely imaginary for crack speeds that exceed the critical value, and can be purely real for trans-sonic speeds.

Examination of Eqs. (41a)–(41c) in light of Appendix C shows that these possibilities exist for the solutions obtained here. Indeed, with the general results (30), (33)–(36), (40) and (44) available, these and other possibilities can be illustrated in terms of more specific situations. Such illustrations are the focus of Part II of this study.

7. Comments

As Part I of a two-part study, extension by a semi-infinite crack in plane strain along the interface of two perfectly bonded dissimilar linearly elastic half-spaces has been treated. Both half-spaces were of a class of orthotropic or transversely isotropic solid, the material symmetry axes were aligned with the interface and its normal, and isotropy was a special case. A steady dynamic situation was considered, in which the crack extended at any constant speed.

Analysis has produced exact formulas for the displacement fields in each half-space for the various ranges of crack speed. These exhibit the possibility of displacement gradient discontinuities that radiate from the crack edge when its speed exceeds the sub-sonic level. Closed-form expressions for the crack-opening displacement gradients and the interface stresses ahead of the crack have also been produced from the solution to coupled singular integral equations. The expressions demonstrate that the eigenvalues of the equations can, depending on crack speed, be complex/imaginary conjugates, purely real, or zero. Such variations are known to control, and perhaps remove, the singular nature of the stresses near the crack edge, and to introduce sign-changes which occur an infinite number of times at the crack edge.

A key step in the solution process has been the factorization of certain quantities that depend on crack speed and material properties. This allows the cancellation of terms that induce both branch points and non-isolated roots in the complex crack speed plane. While these are in general complex, and so are of limited physical interest, the cancellation process puts, despite anisotropy, the solution expressions into simple forms. Moreover, effective Rayleigh and Stoneley functions of crack speed emerge that are more compact than those generally employed in dynamic isotropic studies (Cagniard, 1962; Achenbach, 1973; Brock, 1976).

These functions, in turn, allow compact expressions, analytic to within simple quadratures, for the Rayleigh speeds and, when it exists, the Stoneley speed. The latter was found always to exceed the minimum Rayleigh speed in the two half-spaces. In Table 1, key material properties for five examples of the class of materials treated here—beryl, cobalt, ice, magnesium, titanium (Payton, 1983; Bloor et al., 1994)—are given, as well as the Rayleigh speed v_R, and the Rayleigh speed non-dimensionalized with respect to the corresponding rotational wave speed c_R. The c_R—values are in agreement with data given by Payton (1983).

All possible dissimilar pairs of the materials save ice have been examined, and only cobalt and magnesium found to have a Stoneley speed. Data is presented in Table 2, along with, for comparison, the Stoneley and Rayleigh speeds (v_S, v_R) for two isotropic material combinations (Brock, 1998).

Table 1 indicates that, as fractions of the corresponding rotational wave speeds, the Rayleigh speeds are similar, as is the case for isotropic materials (Sokolnikoff, 1983). Table 2 shows that the cobalt/magnesium combination gives a Stonely speed that lies between the two Rayleigh speeds, while the Stoneley speed exceeds both Rayleigh speeds in the isotropic cases. The closeness of the values in all three situations is noted.

In Part II of this study, the expressions developed here are examined in more detail for two important special cases.

Table 1
Properties of some orthotropic/transversely isotropic materials

	α	β	m	c_{44} (GPa)	c_R	v_R (m/s)
Beryl	3.62	4.11	2.01	68.6	0.956	5825
Cobalt	4.74	4.07	2.37	75.5	0.962	2802
Ice	4.57	4.26	2.64	3.17	0.959	1705
Magnesium	3.74	3.61	2.3	16.4	0.943	2897
Titanium	3.88	3.47	2.48	46.7	0.936	3016

Table 2
Rayleigh and Stoneley speeds

	v_R (m/s)	v_S (m/s)
Cobalt/magnesium	2802/2897	2887
Aluminum[a]/steel[a]	2856/2841	3025
Aluminum[a]/titanium[a]	2856/2825	3027

[a] Treated as isotropic.

Appendix A

In Eq. (24) the formulas

$$\begin{aligned}
\ln G_S &= -\frac{1}{\pi}\left(\int_{n_k}^{n_k\sqrt{\beta_k}} \tan^{-1}\frac{E_k}{F_k} + \int_{n_k\sqrt{\beta_k}}^{n_l} \tan^{-1}\frac{G_{kk}}{F_{kk}} + \int_{n_l}^{n_l\sqrt{\beta_l}} \tan^{-1}\frac{G_l}{H_l}\right)\frac{dt}{t} \\
\ln G_S &= -\frac{1}{\pi}\left(\int_{n_k}^{n_l} \tan^{-1}\frac{E_k}{F_k} + \int_{n_l}^{n_k\sqrt{\beta_k}} \tan^{-1}\frac{E_{kl}}{F_{kl}} + \int_{n_k\sqrt{\beta_k}}^{n_l\sqrt{\beta_l}} \tan^{-1}\frac{G_l}{H_l}\right)\frac{dt}{t} \\
\ln G_S &= -\frac{1}{\pi}\left(\int_{n_k}^{n_l} \tan^{-1}\frac{E_k}{F_k} + \int_{n_l}^{n_l\sqrt{\beta_l}} \tan^{-1}\frac{E_{kl}}{F_{kl}} + \int_{n_l\sqrt{\beta_l}}^{n_k\sqrt{\beta_k}} \tan^{-1}\frac{G_k}{H_k}\right)\frac{dt}{t}
\end{aligned} \tag{A.1}$$

hold for the cases (18a–c), respectively, when $(k = 1, l = 2)$. For the cases (18d–f), respectively, we set $(k = 2, l = 1)$. In Eq. (A.1) the formulas

$$\begin{aligned}
E_k &= 2M_l(1-m_k)B'_k + (a+b)_l(\alpha_l B_l A_k B'_k + \alpha_k B'_k a_k A_l) - \frac{\mu_k}{\mu_l}C_k B'_k (A+B)_l - \frac{\mu_l}{\mu_k}B'_k R_l \\
F_k &= 2M_l A_k + (a+b)_l(\alpha_l B_l a_k A_k - \alpha_k B'_k b'_k A_l) - \frac{\mu_k}{\mu_l}c_k^2 A_k (A+B)_l - \frac{\mu_l}{\mu_k}A_k R_l
\end{aligned} \tag{A.2}$$

$$G_{kk} = 2M_l M'_k - \frac{\mu_k}{\mu_l}(A+B)_l R'_k - \frac{\mu_l}{\mu_k}(A'+B')_k R_l \tag{A.3a}$$

$$H_{kk} = -(a+b)_l(a'+b')_k(\alpha_k B'_k A_l + \alpha_l B_l A'_k) \tag{A.3b}$$

$$G_k = 2A_k M'_l - (a'+b')_l(\alpha_k B'_k a_k A'_l + \alpha_l B'_l A_k b'_k) - \frac{\mu_l}{\mu_k}A_k R'_l - \frac{\mu_k}{\mu_l}c_k^2 A_k (A'+B')_l \tag{A.4a}$$

$$H_k = -2(1-m_k)B'_k M'_l + (a'+b')_l(\alpha_k B'_k b'_k A'_l - \alpha_l B'_l a_k A_k) + \frac{\mu_l}{\mu_k}B'_k R'_l - \frac{\mu_k}{\mu_l}(A'+B')_l C_k B'_k \tag{A.4b}$$

$$\begin{aligned}
E_{kl} = E_{lk} &= 2A_k(1-m_l)B'_l + 2A_l(1-m_k)B'_k + (a_k a_l - b'_k b'_l)(\alpha_k B'_k A_l + \alpha_l B'_l A_k) \\
&\quad - \frac{\mu_k}{\mu_l}(A_l B'_k C_k + B'_l c_k^2 A) - \frac{\mu_l}{\mu_k}(A_k B'_l C_l + B'_k c_l^2 A_l) \\
F_{kl} = F_{lk} &= 2A_k A_l - 2(1-m_k)(1-m_l)B'_k B'_l - (a_k b'_l + a_l b'_k)(\alpha_k B'_k A_l + \alpha_l B'_l A_k) \\
&\quad + \frac{\mu_k}{\mu_l}(B'_l c_k^2 A_k - A_l B'_k C_k) + \frac{\mu_l}{\mu_k}(B'_k c_l^2 A_l - A_k B'_l C_l)
\end{aligned} \tag{A.5}$$

hold, where the definitions (15a)–(15c), (16a)–(16c) and (25) apply.

Appendix B

For $v > v_0$, with $c_0 \pm i0$ understood, Eq. (17) gives

$$\begin{aligned}
S &= \mu_k\mu_l(F_k \mp iE_k), \quad n_k < c_0 < \min(n_k\sqrt{\beta_k}, n_l) \\
S &= \mu_k\mu_l(F_{kl} \mp iE_{kl}), \quad \max(n_k, n_l) < c_0 < \min(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l}) \\
S &= \mu_k\mu_l(F_{kk} \mp iG_{kk}), \quad n_k\sqrt{\beta_k} < c_0 < n_l \\
S &= \mu_k\mu_l(H_k \mp iG_k), \quad \max(n_k, n_l\sqrt{\beta_l}) < c_0 < n_k\sqrt{\beta_k}
\end{aligned} \tag{B.1}$$

where Eqs. (A.2)–(A.5) hold. For $c_0 > \max(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l})$,

$$\frac{S}{\mu_k\mu_l} = -2M_k'M_l' + (a'+b')_k(a'+b')_l(\alpha_k B_k'A_l' + \alpha_l B_l'A_k') + \frac{\mu_k}{\mu_l}(A'+B')_l R_k' + \frac{\mu_l}{\mu_k}(A'+B')_k R_l' \tag{B.2}$$

In Eqs. (B.1) and (B.2), the definitions Eqs. (15a)–(15c), (16a)–(16c) and (25) hold. By letting $k = (1,2)$ when $l = (2,1)$, Eqs. (B.1) and (B.2) cover the six cases Eqs. (18a–c) and (18d–f).

It is noted that the right-hand sides of Eqs. (13a), (13b) and (14) differ only because the roles of the subscripts 1 and 2 are reversed. Therefore, when $v > v_0$, with $c_0 \pm \mathrm{i}0$ understood, the generic formulas

$$\frac{\pm\mathrm{i}mB'}{\psi(a\pm\mathrm{i}b')}\begin{bmatrix} A_U \\ A_V \end{bmatrix}, \quad \frac{\mp\mathrm{i}a}{B'(a\pm\mathrm{i}b')}\begin{bmatrix} -A_U \\ A_V \end{bmatrix}, \quad \frac{\mp\mathrm{i}}{B'(a\pm\mathrm{i}b')}\begin{bmatrix} -B_U \\ B_V \end{bmatrix}, \quad \frac{\pm\mathrm{i}mb'}{\psi B'(a\pm\mathrm{i}b')}\begin{bmatrix} B_U \\ B_V \end{bmatrix} \tag{B.3}$$

hold when $n < c_0 < n\sqrt{\beta}$, where the subscripts $k = (1,2)$ are understood here and on the matrix factors. Similarly, the formulas

$$\frac{mB'}{\psi(b'-a')}\begin{bmatrix} A_U \\ A_V \end{bmatrix}, \quad \frac{\pm\mathrm{i}a'}{B'(b'-a')}\begin{bmatrix} -A_U \\ A_V \end{bmatrix}, \quad \frac{-1}{B'(b'-a')}\begin{bmatrix} -B_U \\ B_V \end{bmatrix}, \quad \frac{\pm\mathrm{i}mb'}{\psi B'(b'-a')}\begin{bmatrix} B_U \\ B_V \end{bmatrix} \tag{B.4}$$

govern when $c_0 > n\sqrt{\beta}$. The coefficient matrices of the elasticities on the right-hand sides of Eqs. (13a) and (13b) take the forms

$$\begin{bmatrix} -P_kA_l(a+b)_l + (1-m_k)b_k'B_k'Q_k & \mp\mathrm{i}m_kb_k'R_l \\ P_kM_l \mp \mathrm{i}b_k'Q_k\alpha_lB_l(a+b)_l & -\psi_kR_l \end{bmatrix}$$
$$\begin{bmatrix} -a_kP_kM_l - B_k^2Q_kA_l(a+b)_l & a_k\psi_kR_l \\ -B_k^2Q_kM_l - a_kP_k\alpha_lB_l(a+b)_l & m_kB_k^2R_l \end{bmatrix} \tag{B.5}$$

respectively, when $n_k < c_0 < \min(n_k\sqrt{\beta_k}, n_l)$, where $(k=1, l=2)$. These are replaced by

$$\begin{bmatrix} -P_kA_la_l + (1-m_k)b_k'B_k'Q_k \pm \mathrm{i}(b_k'A_kQ_k + P_kA_lb_l') & -m_kb_k'(C_lB_l' \pm \mathrm{i}c_l^2A_l) \\ P_kA_l - b_k'Q_k\alpha_lB_l'a_l \pm \mathrm{i}B_l'[b_k'Q_k\alpha_lb_l' - (1-m_l)P_k] & \psi_k(-c_l^2A_l \pm \mathrm{i}C_lB_l') \end{bmatrix} \tag{B.6a}$$

$$\begin{bmatrix} -A_l(a_kP_k + B_k^2Q_ka_l) \pm \mathrm{i}[a_kP_k(1-m_l)B_l' + B_k^2Q_kA_lb_l'] & a_k\psi_k(c_l^2A_l \mp \mathrm{i}C_lB_l') \\ -B_k^2Q_kA_l + a_kP_k\alpha_lB_l'b_l' \pm \mathrm{i}B_l'[B_k^2Q_k(1-m_l) + a_kP_k\alpha_lA_l] & m_kB_k^2(c_l^2A_l \mp \mathrm{i}C_lB_l') \end{bmatrix} \tag{B.6b}$$

when $\max(n_k, n_l) < c_0 < \min(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l})$. The results

$$\begin{bmatrix} b_k'M_k'Q_k - P_kA_l(a+b)_l & \mp\mathrm{i}m_kb_k'R_l \\ P_kM_l \mp \mathrm{i}b_k'Q_k\alpha_lB_l(a+b)_l & -\psi_kR_l \end{bmatrix} \tag{B.7a}$$

$$\begin{bmatrix} -B_k^2Q_kA_l(a+b)_l \pm \mathrm{i}a_k'P_kM_l & \mp\mathrm{i}a_k'\psi_kR_l \\ -B_k^2Q_kM_l \pm \mathrm{i}a_k'P_k\alpha_lB_l(a+b)_l & m_kB_k^2R_l \end{bmatrix} \tag{B.7b}$$

then hold for $n_k\sqrt{\beta_k} < c_0 < n_l$, while the forms

$$\begin{bmatrix} P_kA_l'(a'+b')_l + b_k'Q_k(1-m_k)B_k' \pm \mathrm{i}b_k'Q_kA_k & -m_kb_k'R_l' \\ -P_kM_l \pm \mathrm{i}b_k'Q_k\alpha_lB_l'(a'+b')_l & \pm\mathrm{i}\psi_kR_l' \end{bmatrix} \tag{B.8a}$$

$$\begin{bmatrix} B_k^2Q_kA_l'(a'+b')_l \pm \mathrm{i}a_kP_kM_l' & \mp\mathrm{i}a_k\psi_kR_l' \\ a_kP_k\alpha_lB_l'(a'+b')_l \pm \mathrm{i}B_k^2Q_kM_l' & \mp\mathrm{i}m_kB_k^2R_l' \end{bmatrix} \tag{B.8b}$$

are valid when $\max(n_k, n_l\sqrt{\beta_l}) < c_0 < n_k\sqrt{\beta_k}$. For $c_0 > \max(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l})$, finally, the matrix elements are

$$\begin{bmatrix} b'_k Q_k M'_k + P_k A'_l (a'+b')_l & -m_k b'_k R'_l \\ \pm \mathrm{i}[b'_k Q_k \alpha_l B'_l (a'+b')_l - P_k M'_l] & \pm \mathrm{i}\psi_k R'_l \end{bmatrix}$$
$$\begin{bmatrix} a'_k P_k M'_l + B_k^2 Q_k A'_l (a'+b')_l & -a'_k \psi_k R'_l \\ \pm \mathrm{i}[B_k^2 Q_k M'_l + a'_k P_k \alpha_l B'_l (a'+b')_l] & \mp \mathrm{i} m_k B_k^2 R'_l \end{bmatrix} \tag{B.9}$$

govern. For the right-hand side of Eq. (14), the formulas (B.5)–(B.9) still hold, but now ($k = 2, l = 1$). By properly choosing the subscripts (k, l), the forms (B.3) and (B.4) can be combined with Eqs. (B.5)–(B.9) to identify the real and imaginary parts indicated in Eq. (33) when $v > v_0$ for all the cases (18a–c) and (18d–f) in either half-space 1 ($y > 0$) or half-space 2 ($y < 0$).

Appendix C

For $v > v_0$, with $c_0 \pm \mathrm{i}0$ understood, Eq. (39) yields the results

$$\begin{aligned} N &= \mu_l A_k R_l - \mu_k c_k^2 A_k M_l \mp \mathrm{i}B'_k[\mu_l(1-m_k)R_l - \mu_k C_k M_l] \\ N_A &= \mu_l A_k a_k R_l + \mu_k A_l (a+b)_l c_k^2 A_k \mp \mathrm{i}B'_k[\mu_l A_k b'_k R_l + \mu_k C_k B'_k A_l (a+b)_l] \\ N_B &= -\mu_l \alpha_k B'_k b'_k R_l + \mu_k \alpha_l B_l (a+b)_l c_k^2 A_k \mp \mathrm{i}B'_k[\mu_l \alpha_k A_k a_k R_l + \mu_k C_k \alpha_l B_l (a+b)_l] \end{aligned} \tag{C.1}$$

when $n_k < c_0 < \min(n_k\sqrt{\beta_k}, n_l)$ for the cases (18a–c) by choosing ($k = 1, l = 2$). The set (C.1) is replaced by

$$\begin{aligned} N = {} & A_k A_l(\mu_l c_k^2 - \mu_k c_l^2) - B'_k B'_l[\mu_l C_l(1-m_k) - \mu_k C_k(1-m_l)] \mp \mathrm{i}\mu_l[c_l^2 A_l(1-m_k)B'_k + A_k C_l B'_l] \\ & \pm \mathrm{i}\mu_k[c_k^2 A_k(1-m_l)B'_l + A_l C_k B'_k] \end{aligned} \tag{C.2a}$$

$$\begin{aligned} N_A = {} & \mu_l A_k(c_l^2 A_l a_k - C_l B'_l b'_k) + \mu_k A_l(c_k^2 A_k a_l - C_k B'_k b'_l) \mp \mathrm{i}\mu_l A_k(c_l^2 A_l b'_k + C_l B'_l a_k) \\ & \mp \mathrm{i}\mu_k A_l(c_k^2 A_k b'_l + C_k B'_k a_l) \end{aligned} \tag{C.2b}$$

$$\begin{aligned} N_B = {} & -\mu_l a_k B'_k(c_l^2 A_l b'_k + C_l B'_l a_k) - \mu_k \alpha_l B'_l(c_k^2 A_k b'_l + C_k B'_k a_l) \mp \mathrm{i}\mu_l \alpha_k B'_k(c_l^2 A_l a_k - C_l B'_l b'_k) \\ & \mp \mathrm{i}\mu_k \alpha_l B'_l(c_k^2 A_k a_l - C_k B'_k b'_l) \end{aligned} \tag{C.2c}$$

when $\max(n_k, n_l) < c_0 < \min(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l})$, and by

$$N = \pm\mathrm{i}(\mu_k R'_k M_l - \mu_l R_l M'_k) \tag{C.3a}$$

$$N_A = -\mu_l R_l A'_k(a'+b')_k \mp \mathrm{i}\mu_k R'_k A_l(a+b)_l \tag{C.3b}$$

$$N_B = -\mu_l R_l \alpha_k B'_k(a'+b')_k \mp \mathrm{i}\mu_k R'_k \alpha_l B_l(a+b)_l \tag{C.3c}$$

when $n_k\sqrt{\beta_k} < c_0 < n_l$. The formulas

$$N = \mu_l B'_k[\mu_k C_k M'_l - \mu_l(1-m_k)R'_l] \mp \mathrm{i}A_k(\mu_l R'_l - \mu_k c_k^2 M'_l) \tag{C.4a}$$

$$N_A = -A_k[\mu_l b'_k R'_l + \mu_k c_k^2 A'_l(a'+b')_l] \mp \mathrm{i}[\mu_l A_k a_k R'_l - \mu_k C_k B'_k A'_l(a'+b')_l] \tag{C.4b}$$

$$N_B = -\mu_l \alpha_k B'_k a_k R'_l - \mu_k c_k^2 A_k \alpha_l B'_l(a'+b')_l \pm \mathrm{i}[\mu_l \alpha_k B'_k b'_k R'_l + \mu_k C_k B'_k \alpha_l B'_l(a'+b')_l] \tag{C.4c}$$

hold when $\max(n_k, n_l\sqrt{\beta_l}) < c_0 < n_k\sqrt{\beta_k}$, while for $c_0 > \max(n_k\sqrt{\beta_k}, n_l\sqrt{\beta_l})$, the formulas

$$N = \mu_k R'_k M'_l - \mu_l R'_l M'_k$$
$$N_A = \pm i[\mu_l R'_l A'_k (a' + b')_k + \mu_k R'_k A'_l (a' + b')_l] \tag{C.5}$$
$$N_B = \pm i[\mu_l R'_l \alpha_k B'_k (a' + b')_k + \mu_k R'_k \alpha_l B'_l (a' + b')_l]$$

govern. The corresponding formulas for the cases (18d–f) follow from Eqs. (C.1)–(C.5) by setting ($k = 2, l = 1$) and reversing the signs on the right-hand sides of the formulas for N. In all these formulas, the definitions (15a)–(15c), (16a)–(16c) and (25) apply.

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